CHAPTER 1 Introduction and Probability

1.1.1 What do they have in common?

Finding Shared and Unique Characteristics

Have you ever tried to learn how to play a new game just by watching others play it? Chances are you were watching for patterns in the game such as what each player did to earn points or what actions earned penalties. If you were watching a game of soccer, you might figure out that kicking the ball into the goal earned the team a point. You might also notice that the goalie is the only player allowed to use his or her hands on the field. Describing those patterns could help you describe some of the rules of the game. This type of observation is also called generalizing. When you make an observation about what a set of objects or actions have in common, like the actions that will *always* allow you to score a point, you are making a generalization.

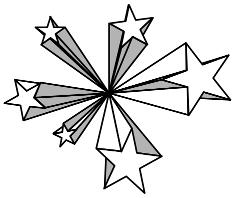
Today you will be working in a team with the goal of making generalizations about characteristics of various sets of objects. As you work, keep the following questions in mind:

What do the items have in common?

What makes the items different?

What are other characteristics that could describe the groups or sets?

1-1. MATH STARS



What do you have in common with your teammates? What makes you unique? Today you will work with your teammates to discover some characteristics that you share and some that make each person in your team different.

Your task: Get a copy of the Lesson 1.1.1A Resource Page and scissors. Fold the resource page along the dotted lines and then cut along the solid lines. Unfold the cutout and glue it onto a piece of paper. Write each team member's name on a star.

- As a team, brainstorm something that all of you have in common that the rest of the class does not already know.
- List your team's common attribute on the paper in the center of the four stars.
- Find ways that each person in the team is unique from the others (things that are true about that person only), and write those things on each person's star.

To help you work together today, each team member has a specific job, assigned by your first name (or by your last name if two individuals have the same first name).

Team Roles

Resource Manager: If your first name comes first alphabetically:

- Make sure your team has a Lesson 1.1.1A Resource Page, scissors, a glue stick, and colored paper.
- Ask the teacher when the *entire* team has a question. *"No one has an idea? Should I ask the teacher?"*
- Make sure your team cleans up by delegating tasks. You could say, *"I will put away the ________ while you _________ ."*

Facilitator: If your first name comes second alphabetically:

- Start the team's discussion of similarities and differences by asking, *"What might we have in common?"*
- Keep everyone discussing each part together by asking questions such as,
  "Does anyone have ideas for what makes us each unique (different)?"
  "What else might we have in common?"

Recorder/Reporter: If your first name comes third alphabetically:

- When your team is called on, share your team's ideas and reasons with the class.
- Help the team agree on an idea: *"Do we agree that this would not be obvious to the rest of the class?"*

Task Manager: If your first name comes fourth alphabetically:

- Remind the team to stay on task and not to talk to students in other teams. You can suggest, *"Let's move on to the next part of the problem."*
- Listen for reasons and ask your teammates to justify their thinking. "*Why do you think that?"* or *"Would this be obvious to the rest of the class?"*

1-2. WHAT DO THEY HAVE IN COMMON?

In mathematics it is sometimes useful to make general statements about sets of objects. In this problem you will be given a set of cards with mathematical objects on them. With your team:

- Determine what each set has in common.
- Record what is written on the cards on your paper.
- Add two new examples that would belong in the set on your paper.
- Describe in words what it is they have in common.

When you are finished with a set of cards, call the teacher to your team. Explain your generalization to the teacher and get another set of cards.

Review & Preview

1-3. **Mathography:** A mathography is a lot like your life history, except that it is focused on the mathematics in your life.

a. Write a letter about yourself to your teacher. The letter will help your teacher get to know you as an individual. The letter should talk about these three general topics: you, you as a student, and you as a math student.

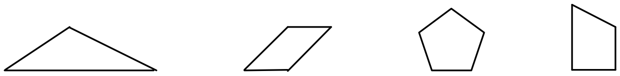
Remember to use complete sentences and make sure that it is neat enough to be read easily. Start the letter with “Dear....” Make sure you sign your letter. This assignment should take 15 to 20 minutes to complete. Parts (b), (c), and (d) have suggestions for what to write about each of the three topics.

b. **You:** Introduce yourself using the name you like to be called. Describe your hobbies, talents, and interests. State your goals or dreams. What are you proud of? What else would you like to share?

c. **You as a Student:** State the importance of school in your life. Describe yourself as a student. What kinds of classroom activities do you do best? What kinds of activities do you find frustrating? Explain which subject(s) is/are your favorites. Tell why you like it (them). How often do you finish in-class assignments? How faithfully do you do your homework?

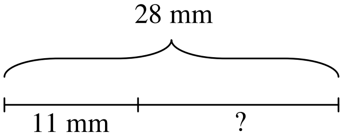
d. **You as a Math Student:** Describe your most memorable moment in math and explain why you remember it. State your favorite math topic. Name your least favorite. Explain how you feel about math this year.

1-4. Decide what the shapes below have in common. Write your answer in a complete sentence. Then draw two more shapes that belong to the same set.

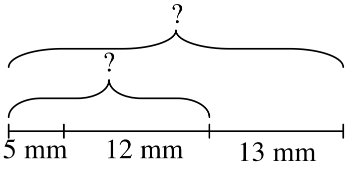


1-5. According to the diagrams below, how long is each unknown piece?

a.



b.



1.1.2 What is the best strategy?

Analyzing a Game

Have you ever watched people win a game again and again? Do you think these people just have good luck? In many cases, winners have a strategy that increases the chance that they will win. How can you develop a winning strategy? Today you will start to answer this question, and later in this course you will learn more about analyzing a game.

Throughout this course, it will be important to describe and explain your ideas. As you play a game called Color-Rama, describe any patterns that you notice and explain your thinking about why certain choices might be better than others. Paying close attention to what is happening as you play can give you ideas to help you build a winning strategy.

1-6. COLOR-RAMA

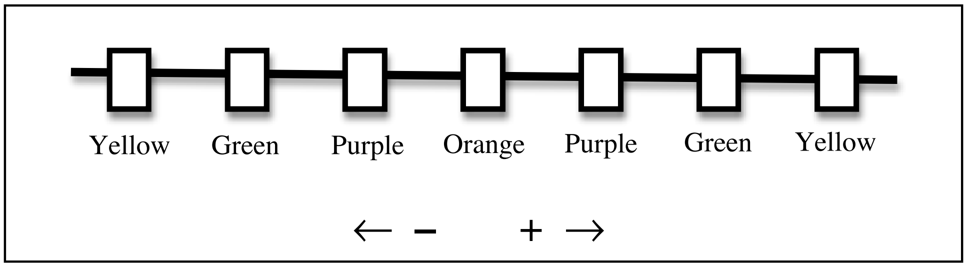
Your teacher will challenge your class to a game of Color-Rama! To play, a marker will be placed on the orange space on the board below. Your class will need to select *one* color for your class and *a different color* for your teacher. Then a volunteer will flip a coin three times. If the coin lands with the "+" showing, the marker will move one space to the right, and if the "–" is showing, then the marker will move one space to the left. If after three flips, the marker is on your class's color, your class wins! If it lands on a color no one picked, then no one wins. Which color should you choose? Is there a way to predict which color the marker will land on?

Before you play, discuss the questions below with your team. When talking about strategies, be sure to describe your ideas and explain your reasoning. When your class has considered all the colors, move on to problem 1-7.

Discussion Points

Does it matter which color is chosen?

Are all the colors equally likely to win? How can you decide?



1-7. LEARNING MORE ABOUT THE GAME

If you want to win, is one color a better choice than the others? Is there a color that you should not pick? One way to answer these questions is to play the game and keep track of what happens.

Your teacher will give you and a partner a Lesson 1.1.2A Resource Page and a coin with sides labeled "+" and "–." You and your partner will need a small object such as an eraser or paper clip to serve as a marker. Once you have all of your supplies, follow the directions below.

a. Play the game several times and be sure to select a different color each time. Keep track of which colors win and lose each time. What do you notice?

b. After you and your partner have played the game at least five times, join with another pair of students to form a team and discuss the following questions:

  - Does the color you choose seem to affect your chances of winning?
  - Is each color choice equally likely to result in a win? Explain why or why not in as many ways as you can.

c. Play the game a few more times. Do your results seem to confirm your answers from part (b) above? Why do your results make sense?

1-8. PLAYING THE GAME

Now is the moment you have been waiting for! As a class, choose two colors (one for your class, one for your teacher) that you think will improve the class's chance of beating the teacher.

a. Is there any color you could choose that would guarantee that you would win this game every time that you played? Explain why or why not.

b. Is there a color that would guarantee that you would *not* win? Explain why or why not.

1-9. What makes a game fair? Discuss this question with your partner and then think about whether there is a way to change the rules of Color-Rama to make it a fair game. Decide on any changes to the rules that you would recommend.

a. Play the game a few times with your new rules. Be prepared to describe to the class the changes you made and explain your reasons for making the changes.

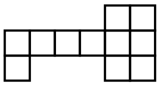
b. Is your new game fair? If not, could you make it fair? Work with your team to find a way to explain how you know your game is fair or why you cannot make it fair.

METHODS AND MEANINGS

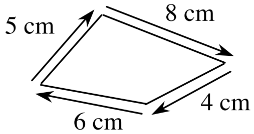
MATH NOTES

Perimeter and Area

The **perimeter** of a shape is the total length of the boundary (around the shape) that encloses the interior (inside) region on a flat surface. See the examples at right.



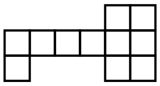
Perimeter = 20 units



Perimeter =

$5 + 8 + 4 + 6 = 23\,\text{cm}$

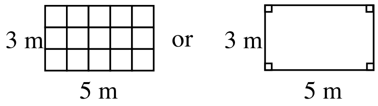
The **area** is a measure of the number of square units needed to cover a region on a flat surface. See the examples at right.



Area = 11 sq. units

The **area of a rectangle** is found by multiplying the lengths of the base and height. See the examples at right.

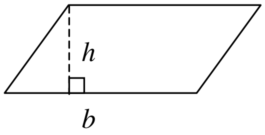
$$A = b \cdot h$$



Area = $5 \cdot 3 = 15\ \text{m}^2$ (square meters)

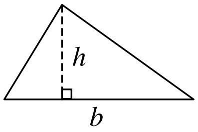
The **area of a parallelogram** is equal to a rectangle with the same base and height. If the base of the parallelogram is length b and the height length h, then the area of the parallelogram is:

$$A = b \cdot h$$



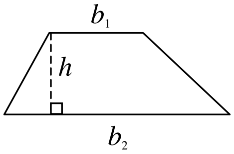
The **area of a triangle** is half the area of a parallelogram with the same base and height. If the base of the triangle is length b and the height length h, then the area of the triangle is:

$$A = \frac{1}{2} b \cdot h$$



Finally, the **area of a trapezoid** is found by averaging the two bases and multiplying by the height. If the trapezoid has bases b_1 and b_2 and height h, then the area is:

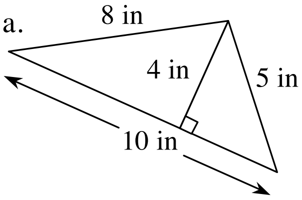
$$A = \frac{1}{2}(b_1 + b_2)h$$



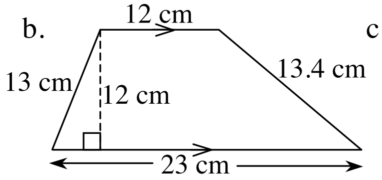
Review & Preview

1-10. Find the perimeter and area of each figure below. Review the Math Notes box in this lesson for help. Be sure to include the correct units in your answers.

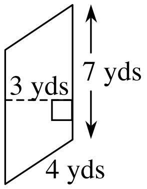
a.



b.



c.



1-11. Think about games that you can remember playing with your friends or family.

a. Talk with your family and/or friends and list as many games as you can think of.

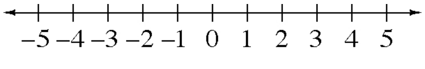
b. Label each game on your list as “fair,” “unfair,” or “I’m not sure.” For any game that you think is unfair, write down your reasons.

1-12. For each of the following pairs of events, predict which is more likely and explain your reasoning.

a. Event 1: You will win more than $1,000,000 in a lottery sometime in your life.
Event 2: You will learn to speak a language that you do not already know.

b. Event 1: Your classmates will all wear the same kind of shoes on the same day.
Event 2: Your classmates will all eat the same thing for lunch on the same day.

1-13. Copy the number line below onto your paper. Place a point on the number line and label the point for each of the following numbers: -4 , 3, 0.5, -3.5 , 0, 1, -2 , 5.



1-14. Maria was playing a game with her brother. She said, *"I'm thinking of a number. When you multiply my number by six and add seven you get 25. What is my number?"*

a. Find Maria's number.

b. Explain how you figured out your answer to Maria's number puzzle.

1-15. Maria's brother came back with another problem. He said, *"If you multiply my number by 9 and add 4 you get 67."*

a. What number was he thinking of ?

b. Explain how you figured out your answer to Maria's brother's number puzzle.

1.1.3 What is my number?

Finding Unknowns

Today you will think about how to find a mystery number based on information given to you as part of a game. You will practice mental math computations and investigate ways to represent the situation as you try to solve the problem. When you solve these puzzles, you might need to reverse your thinking. Reversing is an important way of thinking mathematically. It means solving problems both backwards and forwards. As you work with your team, ask the following questions:

How can we represent it?

What is the best approach for this problem?

Have we found all of the answers?

1-16. GUESS MY NUMBER

Today you will play the "Guess My Number" game. You will need a pencil and a piece of paper. Your teacher has thought of a number and will tell you some information about it. Your task is to figure out what your teacher's number is. (You can use your paper if it helps.) Then explain why you think it is the mystery number.

Game #1: *When I triple my number and add five, I get eleven. What is my number?*

Game #2: *When I add two to my number and then multiply it by five, I get thirty. What is my number?*

Game #3: *When I take half of my number and add two, I get twenty-four. What is my number?*

Game #4: *When I double my number and add eight I get my number plus twelve. What is my number?*

Game #5: *When I double my number, add four, and then subtract my number and subtract three, I get my number plus one. What is my number?*

1-17. Make up your own Guess My Number games with your team and be ready to share them with the class. Make at least one game that has only one answer and one that works for all numbers. Can you create a game that works for no numbers?

MATH NOTES

METHODS AND MEANINGS

Mean

To understand a set of data, you often need to be able to describe the approximate "center" of that data. One way to do this is to find the **mean** of the data set, which is also called the **arithmetic average**.

To find the mean of a set of data, add the values of the data elements (numbers) and then divide by the number of items of data. The mean is a useful way to describe the data when the set of data does not contain **outliers**, that is, numbers that are much smaller or much larger than most of the other data in the set.

Suppose the following data set represents the number of home runs hit by the best seven players on a major league baseball team:

16, 26, 21, 9, 13, 15, and 9.

The mean is $\frac{16+26+21+9+13+15+9}{7} = \frac{109}{7} \approx 15.57$.

This number shows that a typical player among the best seven home run hitters on the team hits about 15 or 16 home runs each season.

Review & Preview

1-18. Thu wants to play "Guess My Number." She states, "*When I triple my number and add 5, I get 26. What is my number?*" What is her number? Show how you know.

1-19. One of the subjects that you will study in this course is how to describe a set of data. One of the ways that you may have seen before is finding an **average** (also called a **mean**). Review what an average is, as well as how to find it, in the Math Notes box for this lesson. Then find the average (mean) for each set of data below.

a. Jane's quiz scores: 82, 64, 73, 91, 85

b. The number of cats your teammates have as pets: 0, 1, 3, 2

c. The number of minutes Pam talked on the phone: 35, 40, 12, 16, 25, 10

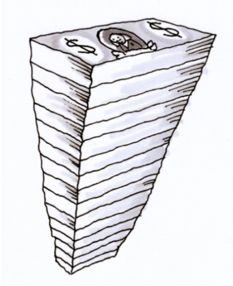
1-20. Imagine that you had one million brand new one-dollar bills. Make a guess about how much space they would need. Would these bills fit in your wallet? Would they fit in a suitcase? Would they fit in your bedroom? In your house? Come up with a guess about how much space you would need for one million one-dollar bills and explain your reasoning.

1-21. Julio is an architect who designs skyscrapers. If each story (floor or "layer") of a new building is 15 feet high, help Julio answer the following questions.

a. How high would a two-story building be? What about a 10-story building? What about a 30-story building?

b. If Julio had to design the building to be 750 feet tall, how many stories should the building have?

1-22. According to the U. S. Treasury, a stack of 233 one-dollar bills is one inch high. One bill is about three inches wide and about six inches long. Will one million dollars fit in your classroom? Answer the following questions to find out.



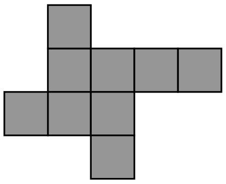
a. If your classroom is 10 feet high, how many dollar bills in one stack will it take to reach the ceiling? Show your work.

b. If the length of your classroom is 30 feet, how many dollar bills (arranged in lengthwise stacks along the floor of one wall) will it take to cover an entire wall?

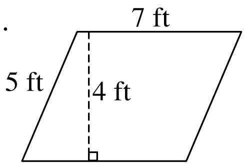
c. If the dimensions of your classroom are 30 feet by 20 feet by 10 feet, how many dollar bills will it take to fill the room? Show all of your work.

1-23. Find the perimeter and area of each figure below.

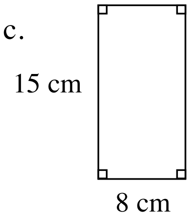
a.



b.

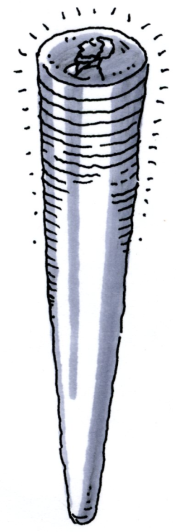


c.



1.1.4 How big is a million?

Investigating a Proportional Relationship



Can you imagine a tower built out of one million pennies? How tall would it be? In this lesson, you will work with your team to predict just how tall such a tower would be. In order to make sense of this question, you will measure some pennies and investigate the relationship between the height of a tower of pennies and the number of pennies in that tower. This is an example of a special relationship, called a **proportional relationship**, which you will learn more about later in this course.

1-24. How tall would a tower of a million pennies be? Would it be taller than your school building? Would it be taller than Mount Everest? (Mount Everest is approximately 8,848 meters or 29,029 feet high.)

Discuss these questions with your team and make a prediction. Record your prediction on the chart provided by your teacher.

1-25. When you are working with your team to solve the "one million pennies" puzzles, as well as other problems in this course, it will be important to work effectively with other people. Effective math conversations are a valuable part of the learning process throughout this course. Choose a member of your team to read aloud these Collaborative Learning Expectations:

COLLABORATIVE LEARNING EXPECTATIONS

Working with other students allows you to develop new ways of thinking about mathematics, helps you learn to communicate about math, and helps you understand ideas better by having to explain your thinking to others. The following expectations will help you get the most out of working together.

T Together, work to answer questions.
E Explain and give reasons.
A Ask questions and share ideas.
M Members of your team are your first resource.
S Smarter together than apart.

1-26. TINY TOWERS

To begin to investigate this question, start by collecting data.

a. How many pennies does it take to build a tower that is one centimeter tall? Use the tools provided by your teacher to find out.

b. On your own paper, create a table like the one at right. Work with your team to complete the missing information. Be prepared to explain your reasoning to the class.

Height of tower (cm)	# of Pennies
1	
2	
3	
4	
5	
…	
10	
…	
20	
…	
30	
…	
100	

1-27. THE HUNDRED-PENNY TOWER

"I have an idea!" Carol said. *"If I know how tall a tower of 100 pennies would be, maybe that can help me figure out how tall a tower of 1,000,000 pennies would be."*

a. Discuss this idea with your team. How could Carol's idea work?

b. Work with your team to figure out how tall a tower of 100 pennies would be. Can you find more than one way to figure this out? Be sure that each member of your team is prepared to explain your thinking to the class.

Height of tower (cm)	# of Pennies
1	
2	
3	
4	
5	
…	…
	100

1-28. THE MILLION-PENNY TOWER

Now it is time to answer the big question: How tall would a tower of a million pennies be?

Your task: Work with your team to calculate the height of a tower of 1,000,000 pennies as accurately as you can. Can you find the height more than one way? Be prepared to explain your ideas to the class.

Discussion Points

Where would 1,000,000 pennies belong in our table?

How can the height of the 100-penny tower help us?

What is the relationship between the number of pennies and the height of the tower?

Further Guidance

1-29. Carol said, *"In order to use my 100-penny height to find the height of 1,000,000 pennies, I have to know how many 100's are in 1,000,000."*

a. How many 1,000's are in 1,000,000? How do you know?

b. How many groups of 100 pennies are in 1,000 pennies?

c. Now work with your team to figure out how many towers of 100 pennies it would take to build a tower of 1,000,000 pennies. Be prepared to explain your reasoning to the class.

d. Use this result along with the height of a 100-penny tower to find the height of a tower of a million pennies.

1-30. Anouk was working with Carol. *"I have another way,"* she said. *"I can see in the table that the number of pennies is always 7 times the number of centimeters. How can we reverse that to find the number of centimeters if we know the number of pennies?"*

a. Discuss Anouk's questions with your team. How can you find the height of the tower if you know the number of pennies in it?

b. Find the height of a tower of 1,000,000 pennies.

Further Guidance
section ends here.

1-31. Anouk wants to compare the million-penny tower to the height of Mount Everest. She read on the Internet that Mount Everest is approximately 8848 meters tall. Her calculation for the height of the million-penny tower is in centimeters. How can she change the units so that she can compare them?

a. Discuss this with your team and then compare your own calculation to the height of Mount Everest.

b. Look at the predictions made by your class at the beginning of this lesson and decide which team came closest to the actual height.

1-32. How accurate is your result? Is there any reason to believe that there may be some amount of error in your calculation? How much do you think this error would matter? Be prepared to share your ideas with the class?

1-33. **Challenge:** How many pennies would be in a tower that is 10 miles high?

1-34. **Additional Challenge:** The Taipei-101 is the second-tallest building in the world. There is a staircase up to the 91st floor with an average of 22 steps from one floor to the next.

a. How many steps would you have to climb to get from the 1st floor to the 91st floor?

b. When you are standing on the 91st floor, you are 1285 feet above the ground. About how many feet tall is each story of the building?

c. About how high is each step?

1-35. LEARNING LOG

Throughout this course, you will be asked to reflect on your understanding of mathematical concepts in a Learning Log. Your Learning Log will contain explanations and examples to help you remember what you have learned throughout the course. It is important to write each entry of the Learning Log in your own words so that later you can use your Learning Log as a resource to refresh your memory. Your teacher will tell you where to write your Learning Log entries. Remember to label each entry with a title and a date so that it can be referred to later.

For your first entry, consider what you know about proportional relationships. The relationship between the number of pennies in a stack and the height of that stack is an example of a proportional relationship. Talk with your team about how you can describe this relationship.

Then record your ideas in your Learning Log, using numbers, words, and tables to help show your thinking. Title this entry “Beginning to Think About Proportional Relationships” and label it with today’s date.

MATH NOTES

METHODS AND MEANINGS

Median

The mean is a useful way to find the center when data values are close together or are evenly spaced. Another tool, **median**, also locates the approximate "center" of a set of data in a different way.

The **median** is the middle number in a set of data arranged numerically. If there are an even number of values, the median is the mean of the two middle numbers. The median is more accurate than the mean as a way to find the center when there are outliers in the data set.

Example: Suppose the following data set represents the number of home runs hit by the best seven players on a major league baseball team:

16, 26, 21, 9, 13, 15, and 9.

The median is 15, since, when arranged in order (9, 9, 13, 15, 16, 21, 26), the middle number is 15.

Mean and median are called **measures of central tendency** because they each describe the "center" of a set of data, but in different ways.

Review & Preview

1-36. In the Math Notes box for this lesson, read and review the information about how to find the median of a data set. Use that information to find the median for Andy's test scores: 76, 84, 93, 67, 82, 87, and 76.

1-37. A stack of six bricks is two feet high. Use that information to answer the questions below.

a. How many bricks are in a stack 20 feet high?

b. How high is a stack of 20 bricks?

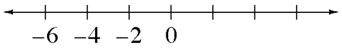
1-38. There are 12 inches in a foot. Use that information to answer the questions below.

a. How many inches tall is a 7-foot basketball player?

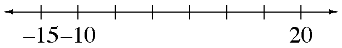
b. If a yard is three feet long, how many inches are in a yard?

1-39. As you can tell from the examples of the number lines below, not all number lines change by one unit from mark to mark. Copy these number lines onto your paper and fill in the missing numbers.

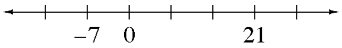
a.



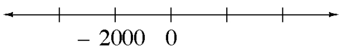
b.



c.



d.

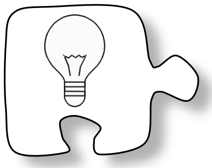


1-40. At the farmers' market, two pounds of peaches cost \$4.20. How much will five pounds cost? Show all of your work or explain your reasoning.

1-41. Janice's mother gave her a ten-dollar bill to buy five pounds each of bananas and apples at the grocery store. When she got there, she found that bananas were 80¢ per pound and apples were \$1.40 per pound.

Did Janice's mother give her enough money? If so, should she receive any change? If not, how much more does she need? Show all of your work.

1.1.5 How can I rewrite it?



Investigating Number Patterns

In the past, you may have looked at number patterns to answer questions or to find the next part of a sequence, but have you ever considered the beauty of mathematics in those patterns? Today you are going to investigate number patterns and learn about equivalent ways to write the same number. As a team, you will share your reasoning and work to justify that two representations are the same. Reasoning and justifying is a way of thinking that you will use often in this course. When you are reasoning and justifying, you will focus on what makes a statement convincing or how you can explain your ideas. As you look at patterns today, ask your team:

What can we predict about the next number in the pattern?

How can we justify our answer?

1-42. In this course, you will transition to using "*x*" as a variable, so this course will avoid using an "*x*" to mean multiplication. The expressions in the problems below use the symbol "$\cdot$" to represent multiplication. Use a calculator to calculate the value of each expression below.

a. $1 \cdot 8 + 1$

b. $12 \cdot 8 + 2$

c. $123 \cdot 8 + 3$

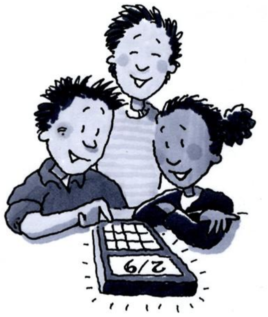
d. $1234 \cdot 8 + 4$

e. What patterns do you see? Discuss them with your team. Be sure that when your team agrees on something, it is recorded on each person's paper.

f. Use patterns you found to *predict* the next three expressions and their solutions. Do not calculate the answers yet. Instead, what do you think they will be?

g. Use your calculator to check the solution for each expression you wrote in part (f). Were your predictions correct? If not, look at the pattern again and figure out how it is changing.

1-43. Sometimes patterns are not created with addition and multiplication, but with the numbers themselves. For example, when the fractions in the sequence below are changed to decimals, an interesting pattern develops.



$$\frac{1}{9}, \frac{2}{9}, \frac{3}{9}, \text{ and } \frac{4}{9}$$

a. Use your calculator to change each of the fractions above to a decimal. Write each fraction and its equivalent decimal on your paper.

b. Decimals like 0.3333... and the others you found in part (a) are called **repeating decimals** because the digits continue infinitely. Instead of using "..." to show that the numbers repeat, mathematicians write a bar over the digits that repeat, like this: $0.\overline{3}$. It is standard to write the repeating digits just once. For example, $0.2222... = 0.\overline{2}$.

List the next five fractions in the sequence $\frac{1}{9}$, $\frac{2}{9}$, $\frac{3}{9}$, and $\frac{4}{9}$. Predict how they will look rewritten as decimals.

c. Find the decimal equivalents of the five fractions you wrote in part (b) using your calculator. Do they match your predictions? Are there any that are different?

1-44. Are $0.999...$, $0.\overline{9}$, and 1 equal? How do you know? Discuss this with the class and justify your response. Help others understand what you mean as you explain your thinking. A visual demonstration is available at www.cpm.org/students/technology.

1-45. Decimal numbers that have only a finite number of digits such as 2.173 and 0.04 are called **terminating decimals**. Some fractions can be written as terminating decimals, such as the examples below.

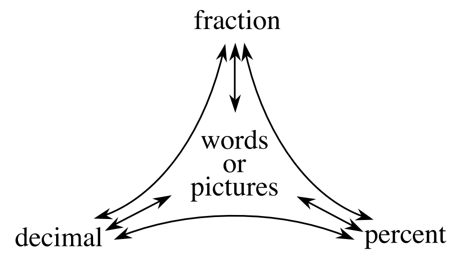
$$\frac{1}{2} = 0.5 \qquad \frac{3}{4} = 0.75$$

Do the decimal equivalents of the numbers below terminate or repeat? Be ready to justify your answer.

a. 0.125 b. $0.\overline{6}$ c. $\frac{5}{6}$

d. 4 e. $\frac{2}{5}$ f. -0.33

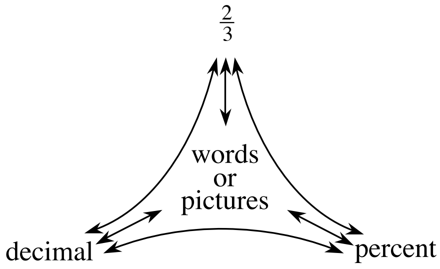
1-46. Representing numbers in multiple ways can help to show what those numbers mean. In problem 1-43, you saw that the fraction $\frac{9}{9}$ (or 1) can be represented as the decimal $0.\overline{9}$, and it can also be represented geometrically with a diagram. Portions can also be represented in words, such as “nine-ninths,” and as **percents**, which are portions of 100. The diagram at right is called the “Portions Web.”



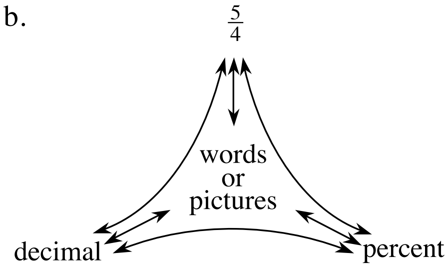
Representations of a Portion

Draw each of the portions webs below on your paper and complete them for the given fractions. In each part, determine if the decimal representation is terminating or repeating.

a.



b.

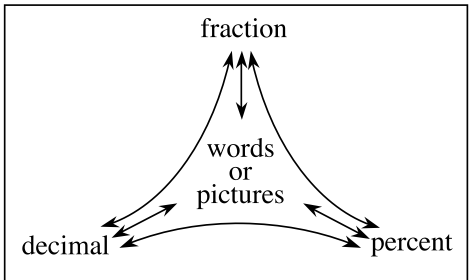


MATH NOTES

METHODS AND MEANINGS

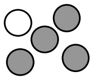
Representations of Portions

The portions web diagram at right illustrates that fractions, decimals, and percents are different ways to represent a portion of a number. Portions can also be represented in words, such as “four-fifths” or “seven-fourths” or diagrams such as those shown below. A complete portions web is shown below right.

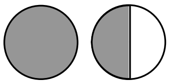


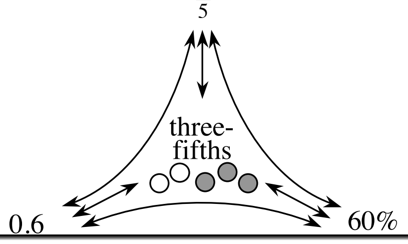
Representations of a Portion

$\frac{4}{5}$ of the objects are shaded:



150% of one circle is shaded:





Review & Preview

1-47. Copy the rows of equations below and write what you *predict* will be the next five rows in the sequence, if the numbers continue to change in the same way.

$1 \cdot 9 + 2 = 11$

$12 \cdot 9 + 3 = 111$

$123 \cdot 9 + 4 = 1111$

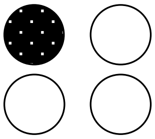
$1234 \cdot 9 + 5 = 11111$

a. What patterns do you see? Write your answer in complete sentences.

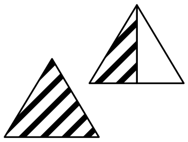
b. Use a calculator to discover if your predictions were correct. If they were not correct, look at the pattern again and figure out how it is changing.

1-48. Look at the representations shown in the Math Notes box for “Representations of Portions.” Copy the diagrams below and write a fraction and a percent for the shaded portion of each one.

a.



b.



c.

1-49. Represent each of these fractions with a diagram and with words.

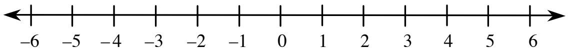
a. $\frac{2}{3}$

b. $1\frac{1}{8}$

c. $\frac{6}{9}$

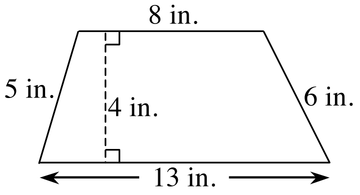
1-50. If five notebooks cost $5.25, how much would three notebooks cost?

1-51. Refer to the number line shown below. List two situations or problems for which you have used a number line in the past.

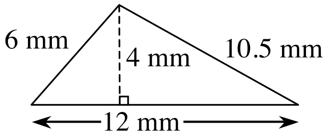


1-52. Find the perimeter and area of each figure below.

a.



b.



1.2.1 What are the chances?

Introduction to Probability

You have probably heard a weather forecaster say that the chance of rain tomorrow is 40%. Have you thought about what that means? Does it mean that it will rain tomorrow for sure? What is the chance that it will not rain? In today's lesson we are going to investigate the chance, or probability, of something happening or not happening. As you do the activities, ask your team the following questions:

What is the probability of the event occurring?

How can we record that probability?

1-53. POSSIBLE OR IMPOSSIBLE?

With your team, make lists of three different types of events:

1. Events that you think are possible but not certain to happen,
2. Events that absolutely certain to happen, and
3. Events that would be impossible to happen.

Be ready to share your lists with the class.

a. Draw a line segment on your paper and label the left end "Impossible" and the right end "Certain."

b. At the "Impossible" end, write the events that your team decided could not happen.

c. At the "Certain" end, write the events your team has decided are certain to happen.

d. Along the line, write the events that you thought were possible. Place them along the line in order from closer to impossible, somewhere in the middle, or closer to certain.

e. Share your information with the rest of the class to be placed on the class line segment.

1-54. GO FISH

Mike wants to win a giant stuffed animal at the carnival. He decided to play the "Go Fish" game, which has three prizes: a giant stuffed animal, a smaller stuffed animal, and a plastic kazoo.

The game is set up with a tank containing 1 green fish, 3 blue fish, and 6 yellow fish. To play, Mike must go fishing. (Note that the game is set up so that every time a player goes fishing he or she will catch a fish.)

To win the giant stuffed animal, Mike needs to catch a green fish.

a. If *all* of the fish in the tank are green, how would you describe the probability of Mike winning a giant stuffed animal?

b. The way the tank is set up (with 1 green, 3 blue, and 6 yellow fish), what are the chances that Mike will catch a black fish?

c. Given the information in the problem, what percent of the time would you expect Mike to catch a green fish and win the giant stuffed animal? Be ready to explain your thinking.

1-55. In the game described in problem 1-54, we could expect Mike to win 10% of the time. A percentage is one way to express the probability that a specific event will happen. You might also have said you expected Mike to win 1 out of every 10 attempts. So the **probability** that Mike will win is $\frac{1}{10}$ because the 1 represents the number of **desired outcomes** (green fish that Mike can catch) and the 10 represents the number of **possible outcomes** (all the fish that Mike could catch).

a. What is the probability that Mike will catch a blue fish? A yellow fish? Write each of these probabilities as a fraction and a percent.

b. We call probabilities such as the one you found in part (a) **theoretical probability** because they are calculated mathematically based upon what we expect.

   What is the theoretical probability of getting a fish of *any* of the three colors, green, blue, or yellow? How do your answers for this problem compare to the probabilities you considered in problem 1-53?

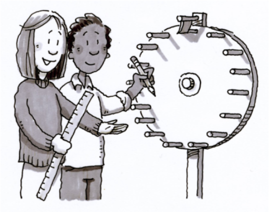
1-56. BUILDING A FISH TANK

The managers of the carnival company have hired your team to help them redesign the "Go Fish" game. They want to control the probability for winning to be sure they make a profit. You can make decisions about the total number of fish in the tank and the colors of those fish to meet the carnival company's requirements.

Work with your team to decide how many fish of each color you could put in the tank so that the probability of catching a green fish will be each of the following numbers. Be ready to explain your thinking to the class.

a. $\frac{2}{7}$ b. $\frac{5}{7}$ c. $\frac{8}{7}$ d. $\frac{7}{7}$

1-57. SPINNERS – THEORY vs. REALITY, Part One



Your teacher will give your team a spinner. You will need to decide how to color the spinner so that it meets the following criteria.

40% should be red. $\frac{1}{10}$ should be yellow.

30% should be blue. The rest should be green.

a. Which color is the most likely result of a spin? How do you know?

b. Which color is the least likely result of a spin? How do you know?

c. Work with your team to determine the theoretical probability of the spinner landing on each of the four colors (red, yellow, blue, and green). Express your answers as fractions and percents.

d. What is the probability of the spinner landing on purple? Explain.

e. What is the probability of the spinner landing on either red or blue?

1-58. SPINNERS – THEORY vs. REALITY, Part Two

Now you will use your new spinner to do an investigation.

a. Each person on your team should spin the spinner 10 times while the other team members record the color resulting from each spin.

b. Write the number of times the spinner landed on each color as the numerator of a fraction with the total number of spins as the denominator.

c. Now combine your team's data with the results from the rest of your classmates and use the class data to write similar fractions as you did in part (b) for each color.

d. The numbers you calculated in part (c) of problem 1-57 are called **theoretical probabilities**. You calculated these numbers (before actually spinning the spinner) to predict what you expected to happen. The numbers you found in your investigation (when you actually spun the spinner) are called **experimental probabilities**, because they are based on the results from an actual experiment or event. Both theoretical and experimental probabilities can be written as a percent, a fraction, or a decimal.

Does it make sense that the theoretical probabilities and the experimental probabilities you calculated for the spinner might be different? Explain.

Does it make sense that the experimental probabilities that you found for the class are different from those found for just your team?

1-59. LEARNING LOG

In this course, you will often be asked to reflect on your learning in a Learning Log. Writing about your understanding will help you pull together ideas, develop new ways to describe mathematical ideas, and recognize gaps in your understanding. Your teacher will tell you where your Learning Log entries should go.

For your first entry, explain how to figure out theoretical and experimental probabilities. Be sure to use examples to make your points clear. Title these notes "Probability" and label them with today's date.

Review & Preview

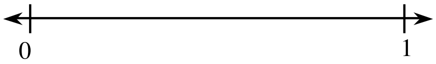
1-60. Joyce's dad packs her lunch and always packs a yogurt. Joyce knows that there are five yogurts in the refrigerator: one raspberry, two strawberry, one blueberry, and one vanilla. Her dad usually reaches into the refrigerator and randomly grabs a yogurt.

a. Which flavor is she most likely to have in her lunch today?

b. What are her chances of finding a vanilla yogurt in her lunch bag?

1-61. Copy the number line below and place the following probabilities on it:

a. A $\frac{1}{4}$ chance that you will be the team member who gets supplies tomorrow.



b. A 25% chance of snow tomorrow.

c. A 0.8 probability of eating vegetables with dinner.

d. P(blue marble) = $\frac{5}{8}$.

e. A 0.01 probability that it will be 85° on Saturday.

1-62. Write "theoretical" or "experimental" to describe the probabilities for each of the following situations.

a. The chance of getting tails when flipping a coin is $\frac{1}{2}$.

b. I flipped a coin eight times and got heads six times, so the probability is $\frac{6}{8}$.

c. My mom packed my lunch three of the past five days, so the probability of my mom packing my lunch is $\frac{3}{5}$.

d. The chance of winning the state lottery is 1 in 98,000,000.

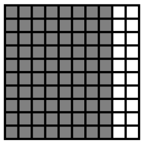
e. Based on mathematical models, the chance of rain today is 60%.

f. Lena got three "hits" in her last seven times at bats, so her chance of getting a hit is $\frac{3}{7}$.

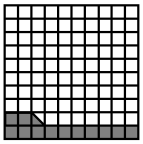
1-63. FRACTIONS AND PERCENTS

Marianna represented several percents as portions of 100 in the pictures below.

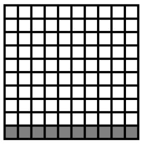
i.



ii.



iii.



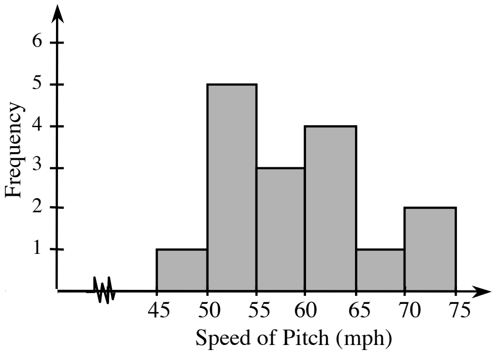
a. Write the percent represented in each picture.

b. Write the portion represented in each picture as a fraction in at least two different ways.

1-64. Find the mean and median for the lengths of the jumping frogs' bodies shown below (the length are in centimeters).

20.3, 12.5, 7.6, 13.9, 9.2, 21.7, 7.6, 17.5, 15.6, 14.1

1-65. Craig is practicing his baseball pitching. He kept track of the speed of each of his throws yesterday, and made the histogram at right.



a. Can you tell the speed of Craig's fastest pitch? Explain.

b. Between what speeds does Craig usually pitch?

c. Based on this data, what is the probability that Craig will pitch the ball between 70 and 75 miles per hour?

1.2.2 How can I use probability to predict?

Investigating Probability

Have you ever tried to predict which football team will win the Big Game? If so, you probably did not just pick the team with the coolest colors or the neatest mascot. You may have based your pick on statistics about win-loss records, player injuries, and other data. Knowing what has happened in the past can sometimes help us predict what will happen in the future. In this lesson, you will use data to make predictions.

As you work with your team to uncover a mystery spinner, keep the following questions in mind:

What is the probability or likelihood?

What do we expect to happen?

How does the actual event compare to our prediction?

What can we know for sure?

1-66. THE MYSTERY SPINNER

Your teacher has a hidden spinner. Your challenge is to perform an experiment that will allow you to predict what the spinner looks like without ever seeing it.

Your task: Your teacher will spin the spinner and announce each result. During the experiment, you will consider several questions about the results and about the hidden spinner, but you will not be allowed to see it. Using the information you get, work with your team to figure out what the spinner looks like. When you think you know, draw a diagram of the spinner.

1-67. Use the data you collected in problem 1-66 to answer the following questions.

a. Based on your data, how can you describe the likelihood of landing on each part of the spinner? How does the spinner that your team drew represent these likelihoods? Be prepared to share your ideas with the class.

b. Use your data to write the experimental probability of each of the following results as a fraction, a decimal, and a percent.

  i. The spinner lands on purple.

  ii. The spinner lands on green or orange.

c. If your teacher were to spin 15 more times, how might this change your answers for part (b)?

d. Do you know for sure that the spinner you drew in problem 1-66 looks exactly like your teacher's? Are you certain that the portions that you drew for each color are the same size as the portions on your teacher's spinner? Why or why not?

1-68. Now your teacher will reveal the mystery spinner.

a. How does your team's spinner compare to the actual spinner? Discuss the similarities and differences.

b. Do your spinner and your teacher's spinner show the same likelihood for each section being spun? Explain why or why not.

1-69. One way to compare your spinner and your teacher's spinner is to calculate the theoretical probability for each colored section.

a. What are some reasons the experimental probability and the theoretical probability for any section of the spinner could be different?

b. Estimate the theoretical probability for getting each color on your teacher's spinner.

c. How do the experimental probabilities (based on your class data) and the theoretical probabilities (based on the actual spinner) compare? How do you think they would compare if there were twice as many spins made? What about three times as many spins?

Problem continues on next page. →

1-69. *Problem continued from previous page.*

d. If you were to spin the spinner the number of times listed below, how many times would you expect it to land on orange? Explain how you found your answers.

i. 6 times

ii. 48 times

e. Approximately how many times would you expect to land on orange if you were to spin 100 times?

1-70. Rachel and her little sister Christie are always arguing about who should wash the dishes at their house. Rachel suggests that they flip coins to solve their argument.

"Flipping a coin is fair, since the head and tail each have an equal chance of happening. Each one has a probability of $\frac{1}{2}$*,"* Rachel says. *"If the coin shows heads, then I do the dishes and if it shows tails then you do them."*

a. Is this system fair? That is, does each girl have an equal likelihood of washing the dishes each day? What is the theoretical probability for heads? For tails?

b. Rachel and Christie flip a coin every evening for the first week. Christie has washed the dishes 4 times, and Rachel has washed the dishes 3 times. Christie tells Rachel that the system is not fair, because Christie has done the dishes more often than Rachel. Is Christie right?

c. After the second week of coin flipping, Christie has washed the dishes 10 times and Rachel has washed the dishes 4 times. Now Christie is really mad at Rachel because she has washed the dishes so many times.

In light of this new information, do you think the system is fair or not fair? What would you recommend to Christie? Discuss your ideas with your team and be prepared to discuss this with the class.

1-71. Flipping a coin has a theoretical probability of $\frac{1}{2}$ for heads and $\frac{1}{2}$ for tails, unless the coin is "rigged." Over the course of a large number of flips, each of the two outcomes will be closer to the theoretical probability ($\frac{1}{2}$) than they will for a few flips. But is it always possible to determine the theoretical probability in every situation?

Look at the situations below and decide with your team for which situations you can find a theoretical probability and for which ones you cannot. If you decide that you can find the theoretical probability, then do so.

a. Picking an Ace from a standard 52-card deck.

b. *Not* rolling a 3 on a standard number cube.

c. The chances of a thumbtack landing point up or on its side.

d. Getting the one red crayon from a set of eight different-color crayons.

e. The likelihood that you will run out of gas on a long car trip.

1-72. **Additional Challenge:** Three friends join together and form a club. They want to randomly select one of themselves as the club's leader. Describe how they can do this fairly using only a coin.

1-73. LEARNING LOG

In your Learning Log, describe the difference between experimental and theoretical probability. Be sure to include examples to illustrate your ideas. Title this "Theoretical and Experimental Probabilities" and label with today's date.

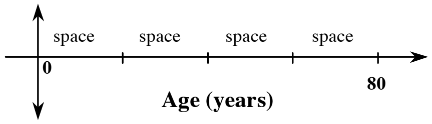
MATH NOTES

METHODS AND MEANINGS

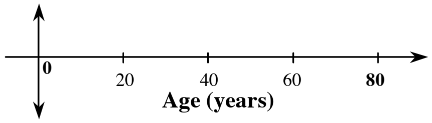
Scaling a Graph

The numbers on the axes of a graph show the **scaling** of the axes. The difference between consecutive markings tells the size of the **interval**. When you scale each axis you must use equal intervals to represent the data accurately. For example, an interval of 5 creates a scale numbered –15, –10, –5, 0, 5, 10, 15 , etc. Unequal intervals distort the relationship in the data.

Notice on the graph at right that 80 marks the end of the *fourth* interval from zero. If you divide 80 years by 4 you can see the length of an interval on this graph is 20.



$$80 \div 4 = 20$$



The second graph at right has each interval labeled. We call labeling the graph "scaling the axis."

Review & Preview

1-74. Imagine that you have a bag containing 10 marbles of different colors. You have drawn a marble, recorded its color, and replaced it fifty times with the following results: 9 purple, 16 orange, 6 yellow, and 19 green marbles. Make a prediction for how many marbles of each color are in the bag. Show all of your work or explain your reasoning.

1-75. A fair number cube with the numbers 1, 2, 3, 4, 5, and 6 is rolled.

a. What is the probability of getting an even number?

b. What is the probability of getting a factor of 6?

1-76. Write “theoretical” or “experimental” to describe the probabilities in each of the following statements.

a. The chance of spinning a four on a spinner numbered one through four is $\frac{1}{4}$.

b. I drew ten cards out of a deck and got hearts three times, so the chance of getting a heart is $\frac{3}{10}$.

c. I made eight out of the last ten free throws, so my chance of making a free throw is 80%.

d. The chance of winning a new car in the raffle is 1 in 32,000.

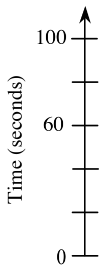
e. Based on mathematical models, the chance of a thunderstorm today is 40%.

1-77. Ramon is saving \$7.75 per week to buy a new cell phone. The model that he has in mind costs \$125.00. Estimate the number of weeks that he will need to save, and then figure out the actual number of weeks.

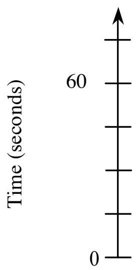
1-78. If 18 inches is equal to $1\frac{1}{2}$ feet, how many feet is a 36-inch board? A 72-inch board? A 144-inch board?

1-79. Read the Math Notes box about scaling axes in this lesson. Then copy these incomplete axes on your paper and write the missing numbers on each one.

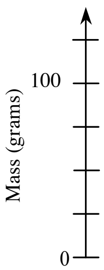
a.



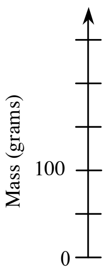
b.



c.



d.



1.2.3 How can I change the chances?

Modifying the Sample Space

Most states have some kind of lottery. How do the chances of winning a lottery when a winner has to match three numbers compare to when the winner has to match seven numbers? How do the chances of catching a particular kind of fish in a pond change just after it is filled with more fish? In this lesson, you will think about the size of the **sample space**, that is, the collection of all possible outcomes of an event. Think about these questions as you work today:

How has the "whole" or total changed?

How has the "portion" or part we are interested in changed?

Has the event become more or less likely?

1-80. Your team will be given a bag containing a set of colored blocks or counters. Each team will receive a bag that is identical to yours.

a. Look at the blocks in your bag. If you were to reach into the bag and select one block without looking, what is the likelihood that it would be:

i.	Red?	*ii.*	Green?
iii.	Blue?	*iv.*	Orange?

b. Do your answers for part (a) represent theoretical or experimental probabilities? Justify your response.

1-81. If you were to select one block from the bag 12 times, replacing the block you drew between each selection, how many of those times would you expect to have selected a blue block? What if you drew 24 times? Discuss both situations with your team and explain your answers.

1-82. DOUBLING BAGS

Now imagine that you and another team have combined the blocks from both of your bags into one bag.

a. Do you think the larger sample space will change the likelihood of drawing different colored blocks? Discuss this with your team and be ready to explain your ideas to the class.

b. Get a second bag of blocks from your teacher and merge the contents of both bags. How many total blocks are there in the bag now? How many are there of each color?

c. Work with your team to find the theoretical probability for selecting each color of block in the combined bags.

d. Has the probability for drawing each different colored block changed? How do your answers for part (c) above compare to the theoretical probabilities that you calculated for the original bag in problem 1-80? With your team, make sense of how the probability for drawing a blue block compares before and after combining the bags.

e. If you were to make 12 draws from the combined bag, again replacing the block between draws, how many times would you expect to draw a blue block? Explain why your answer makes sense.

1-83. In problems 1-80 through 1-82, even though you combined bags and changed the number of selections you made, the probability of drawing a blue block remained the same.

a. Do you think the probabilities would change if you combined three bags? Why or why not?

b. What change do you think you could make in order to increase the chances of choosing a blue block? Explain your reasoning.

1-84. **Additional Challenge:** Use the spinner from the Lesson 1.2.2 Resource Page (available from your teacher or www.cpm.org/students). Play a game with the spinner while keeping score as follows:

- Every time you spin purple, you lose two points.
- Every time you spin green, you get one point.
- Every time you spin orange, you get three points.

a. If you play this game for a long time, do you think it is more likely that you will end up with a positive score or a negative score? Make a prediction and then try it out. You may want to keep score with counters.

b. Change the point values of each color of the spinner so that if you play the game for a long time, it is equally likely that you will have a positive or a negative score.

METHODS AND MEANINGS

MATH NOTES

Probability Vocabulary and Definitions

Outcome: Any result of the action considered, such as rolling a five on a standard number cube or getting tails when flipping a coin.

Event: A desired (or successful) outcome from an experiment, such as rolling an even number on a standard number cube.

Sample Space: All the possible outcomes of a situation. For example, the sample space for flipping a coin is H and T; rolling a standard number cube has six possible outcomes: 1, 2, 3, 4, 5, and 6.

Probability: The likelihood of an event occurring. Probabilities may be written as fractions, decimals, or percents. An event that is guaranteed to happen has a probability of 1, or 100%. An event that has no chance of happening has a probability of 0, or 0%. Events that "might happen" have probabilities between 0 and 1 or 0% and 100%. In general, the more likely an event is to happen, the greater its probability.

Experimental Probability is the probability based on data collected in experiments.

$$\text{Experimental Probability} = \frac{\text{number of successful outcomes in the experiment}}{\text{total number of outcomes in the experiment}}$$

Theoretical Probability is a calculated probability based on the possible outcomes when they all have the same chance of occurring.

$$\text{Theoretical Probability} = \frac{\text{number of successful outcomes (events)}}{\text{total number of possible outcomes}}$$

By "successful" we usually mean a desired or specified outcome (event), such as rolling a 3 on a number cube ($\frac{1}{6}$). To calculate the probability of rolling a 3, first figure out how many possible outcomes there are. Since there are six faces on the number cube, the number of possible outcomes is 6. Of the six faces, only one of the faces has a 2 on it. Thus, to find the probability of rolling a 2, you would write:

$$P(2) = \frac{\text{number of ways to roll 2}}{\text{number of possible outcomes}} = \frac{1}{6} \text{ or } 0.1\overline{6} \text{ or approximately } 16.7\%.$$

Probabilities are written like this:

The probability of pulling a king out of a deck of cards, P(king), is $\frac{4}{52} = \frac{1}{13}$ since there are four kings and 52 cards.

Review & Preview

1-85. Tom keeps all his favorite marbles in a special leather bag. Right now the bag holds five red marbles, four blue marbles, and three yellow marbles.

a. If he randomly chooses one marble to give to a friend, what is the probability that it is blue?

b. Tom does not really want to give away blue marbles and would like to change the probability that he chooses a blue marble to $\frac{1}{10}$. How many marbles that are not blue could he add to the bag so that the probability of choosing a blue marble becomes $\frac{1}{10}$?

1-86. Your team is in charge of games at the CPM Amusement Park. One of the games involves a robotic arm that randomly grabs a stuffed animal out of a large bin. You need to set up the game so that the probability of a customer grabbing a teddy bear is exactly $\frac{1}{2}$.

a. How would you set up the bin? Explain.

b. What if you returned to check on the bin and found that there were four teddy bears left and 12 other animals. What could you add to or remove from the bin to return the probability of selecting a teddy bear to $\frac{1}{2}$?

1-87. A rectangular park is 150 yards on one side and 125 yards on the other.

a. If Debbie walks around the park two times, how far does she walk? Sketch a figure and show your work.

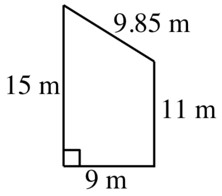
b. If Debbie wanted to walk 1,000,000 yards, how many times would she have to walk around the park?

1-88. If five songs can be downloaded for \$6, what is the cost of seven songs?

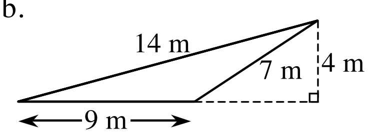
1-89. Richard doubled a whole number and added two. His answer was a whole number less than 20. What numbers could Richard have used?

1-90. Find the perimeter and area of each figure.

a.



b.



1.2.4 How can I change fractions to percents?

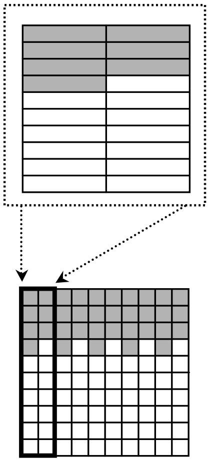
Expressing Fractions as Percents

1-91. After 25 flips of a fair coin, Lorraine and her partner recorded 14 heads ($\frac{14}{25}$ heads).

a. Is this more than, less than, or equal to 50%, the theoretical probability of flipping heads? Talk with your partner about how you know.

b. Percentages are one way to compare different portions of the total. If you have not already done so, work with your partner to calculate what percentage of Lorraine's flips were heads. Be prepared to share your strategy for finding the percentage with the class.

1-92. DaMarr also kept track of the number of heads he flipped. He wrote his experimental probability of flipping heads as the fraction $\frac{7}{20}$. He wants to rewrite $\frac{7}{20}$ as a percent. To do that, he knows he needs to rewrite it as a portion of 100. He started by drawing the picture at right to represent $\frac{7}{20}$:



New diagram

a. As he looked at his picture, DaMarr realized that he could adjust his drawing to show 100 parts instead of 20. DaMarr then drew a new diagram below his original picture. How many of the 100 parts are shaded? Explain your reasoning.

b. Explain how DaMarr can use his new picture to write $\frac{7}{20}$ as a percent.

c. Use a similar strategy to rewrite $\frac{3}{10}$ as a percent. Show your steps.

1-93. Randy was playing "Double Heads" where he would flip two coins at a time. He recorded the number of times he flipped two heads at the same time. He flipped the coins 50 times and there were 20 times when both coins landed heads up, which he recorded as $\frac{20}{50}$.

Note: This stoplight icon will appear periodically throughout the text. Problems with this icon display common errors that can be made. Be sure not to make the same mistakes yourself!

a. How does $\frac{20}{50}$ compare to 50%?

b. Randy claims, "$\frac{20}{50}$ is equivalent to $\frac{70}{100}$ *because if I add 50 to the denominator to get 100 and add 50 to the numerator to get 70, the fraction becomes* $\frac{70}{100}$." Explain to Randy why $\frac{20}{50}$ and $\frac{70}{100}$ are not equivalent portions.

1-94. Three students invented a game in which they flip coins for a minute then determine who flipped the highest percentage of heads. After their first round, each of them thinks that they won the game. Below is what they reported:

- "I think I won," said Maria, "Of my flips $\frac{12}{25}$ were heads."
- Autymn said, "I flipped my coin 40 times and had a total of 18 heads. Since both of my numbers are larger than yours, I must have won."
- Kumar reported, "I recorded 44% of my flips as heads."

Help the students determine their percentage of heads. Justify your answer.

1-95. Change these fractions to percents using DaMarr's strategy from problem 1-92.

a. $\frac{13}{20}$　　b. $\frac{7}{10}$　　c. $\frac{12}{25}$　　d. $\frac{9}{30}$

Review & Preview

1-96. Rewrite each fraction as a percent and each percent as a fraction. Show your thinking with pictures or labeled calculations.

a. $\frac{2}{5}$ b. 45% c. 120% d. $\frac{21}{40}$

1-97. Jessica wants to make a spinner that has all of the following characteristics. Sketch a possible spinner for Jessica. Be sure to label the sections of the spinner with a name and their theoretical probability.

- Blue, red, purple and green are the only colors on the spinner.
- It is half as likely to land on blue as to land on red.
- It is three times as likely to land on purple as green.
- There is a 50% probability of landing on either blue or red and a 50% probability of landing on either purple or green.

1-98. Calculate the mean of each data set below. Can you find any shortcuts that allow you to find the mean without having to do a lot of calculation?

a. 6, 10, 6, 10 b. 11, 12, 12, 13, 12 c. 0, 5, 4, 8, 0, 7

1-99. If five slices of pizza cost $5.50, how much do two slices cost? Ten slices? Half a slice?

1-100. Alice said, "*I am thinking of a number. If you triple my number and subtract 11, you get my number plus 33. What's my number?*" Find Alice's number. Show how you know your answer is correct.

1-101. Find a value for x that will make each equation true.

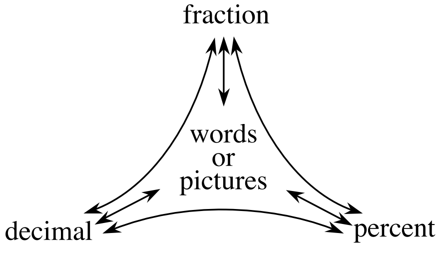
a. $x + 8 = 21$ b. $x - 32 = 55$ c. $3x = 54$ d. $\frac{x}{5} = 10$

1.2.5 How can I represent it?

Rewriting Fractions

When probabilities and portions are expressed as fractions, they can sometimes be difficult to compare. For example, is a $\frac{2}{5}$ probability of winning a game greater than a $\frac{3}{7}$ probability?

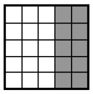
In Lesson 1.1.5, you used different ways to represent a portion to help figure out if $0.\overline{9}$ is equal to 1. In this lesson, you will develop strategies for rewriting fractions in different forms in order to compare different portions.



Representations of a Portion

1-102. FRACTION TO PERCENT

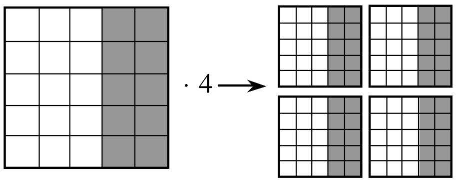
Lila wants to rewrite $\frac{10}{25}$ as a percentage. She decided to represent $\frac{10}{25}$ first in a picture.



a. How many sets of 25 will she need to make 100?

b. How many sets of shaded squares will she need? How many shaded squares is that in total?

c. Lila drew the picture below, and then wrote the equation to the right to represent her work with the shapes:



$$\frac{10}{25} \underset{\cdot 4}{\overset{\cdot 4}{=}} \frac{40}{100}$$

When Lila multiplies the number of shaded squares by 4 and the number of total squares by 4, did the amount of shading compared to the total squares change? Explain how you know.

1-103. Multiplying any number by one does not change the value of the number; this is called the **Identity Property of Multiplication**. When the numerator (top number) and denominator (bottom number) of a fraction are multiplied by the same number, some people like to say the fraction is multiplied by a **Giant One**. For example:

$$\frac{10}{25}\cdot\boxed{\frac{4}{4}}=\frac{40}{100}$$

a. Why is the fraction $\frac{4}{4}$ called a Giant One?

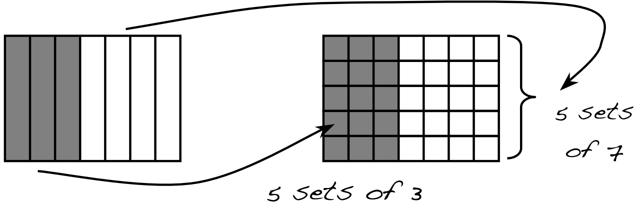
b. Use a Giant One to rewrite each fraction by multiplying each expression below.

i. $\frac{9}{25}\cdot\boxed{\frac{6}{6}}$ *ii.* $\frac{6}{15}\cdot\boxed{\frac{3}{3}}$

c. What if you do not know which numbers to use in a Giant One? Copy and complete each problem below on your paper.

i. $\frac{5}{7}\cdot\boxed{\frac{\ }{\ }}=\frac{40}{\ }$ *ii.* $\frac{5}{7}\cdot\boxed{\frac{\ }{\ }}=\frac{\ }{70}$

1-104. Deanna is trying to compare $\frac{3}{7}$ to $\frac{2}{5}$. She started by representing $\frac{3}{7}$ in a picture, then she divided the picture into five horizontal rows to rewrite the portion as $\frac{15}{35}$. Here is her work so far:



$$\frac{3}{7}\cdot\boxed{\frac{5}{5}}=\frac{15}{35}$$

a. Rewrite $\frac{2}{5}$ as a portion of 35. Is $\frac{2}{5}$ bigger or smaller than $\frac{3}{7}$?

b. Why does rewriting the fractions make it easier to compare the two fractions?

c. Why did Deanna choose 35 for her new denominator? Could she have chosen a different number?

1-105. FAMILIES OF FRACTIONS

Using a Giant One, any fraction can be rewritten as a different fraction that represents the same portion of a whole. Fractions that are equal but are written in different ways are called **equivalent fractions**.

Make three new fractions that are equivalent to each fraction below.

a. $\frac{7}{12}$ b. $\frac{3}{10}$ c. $\frac{9}{27}$

1-106. Adele, Karla, and Lisa are reading the same book, and they have each read a different number of pages.

Adele has read $\frac{5}{8}$ of the book.

Karla has read $\frac{12}{16}$ of the book.

Lisa has read $\frac{13}{24}$ of the book.

Help them decide who has the most left to read.

a. What denominator could they use to compare the portions they have read?

b. Which girl has the largest part left to read? Justify your ideas.

c. How many pages could the book have? Be ready to explain your answer.

1-107. Use the Giant One to rewrite each fraction as a part of 100. Then write the equivalent percentage.

a. $\frac{3}{4} \cdot \boxed{\frac{\;}{\;}} = \frac{\;}{100}$ b. $\frac{42}{200} \cdot \boxed{\frac{\;}{\;}} = \frac{\;}{100}$ c. $\frac{7}{16} \cdot \boxed{\frac{\;}{\;}} = \frac{\;}{100}$

d. How do you decide which number to use in the Giant One?

1-108. Which of the events below is most likely to happen? Justify your answer by rewriting the portion in a different form. In each case, show your thinking with pictures or labeled calculations.

a. A $\frac{4}{5}$ chance of the teacher assigning homework today.

b. A 78% chance of a thunderstorm tomorrow.

c. A $\frac{7}{10}$ probability of picking a green marble.

1-109. LEARNING LOG

Think about which methods you have for comparing two probabilities to decide which is greater. In your Learning Log, create an example and explain two different strategies for comparing them. Title this entry "Comparing Probabilities," and label it with today's date.

MATH NOTES

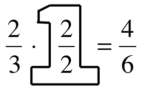
METHODS AND MEANINGS

Additive and Multiplicative Identities

If any number or expression is multiplied by the number "one," the number or expression does not change. The number "one" is called the **multiplicative identity**. So, for any number *x*:

$$1\cdot x = x\cdot 1 = x$$

One way the multiplicative identity is used is to create equivalent fractions using a Giant One.



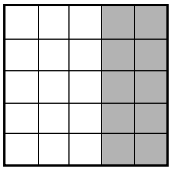
$$\frac{2}{3}\cdot\frac{2}{2}=\frac{4}{6}$$

By multiplying a fraction by a fraction equivalent to one, a new equivalent fraction is created.

The number "zero" is the **additive identity** since adding zero does not change the value or expression.

Review & Preview

1-110. In problem 1-102, Lila rewrote $\frac{10}{25}$ as $\frac{40}{100}$ and represented it with the picture at right. Tony thinks this fraction is also equal to $\frac{2}{5}$.



a. Is Tony correct? Use the picture or calculations to explain your reasoning. Write your answer in complete sentences.

b. How could Tony write an equivalent (equal) fraction using tenths? That is, what fraction in the form $\frac{?}{10}$ can represent the diagram above?

1-111. Zaria wants you to solve this puzzle: *"I am thinking of a number. If you divide my number by 2 and subtract 4, you will get 2. What is my number?"* Show how you know your answer is correct.

1-112. Marissa is drawing coins from a bag that contains 5 pennies, 4 nickels, 5 dimes, and 2 quarters.

a. What is the probability that she will draw a nickel?

b. If one penny, two dimes, and one quarter are added to the bag, what is the new probability that she will draw a nickel?

c. In which situation is it more likely that she will draw a nickel?

1-113. Rewrite each fraction below as an equivalent fraction and as a percent.

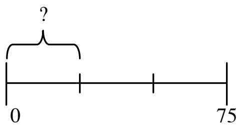
a. $\frac{5}{20}$ b. $\frac{9}{25}$ c. $\frac{9}{6}$

1-114. Rewrite each expression as a single fraction.

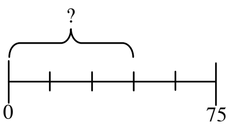
a. $\frac{3}{8}+\frac{1}{8}$ b. $\frac{4}{5}+\frac{3}{5}$ c. $\frac{6}{7}-\frac{2}{7}$

1-115. What is the length of the marked portion of each line segment? Copy the segment onto your paper before finding the missing length. Assume the entire line segment is subdivided into equal sections.

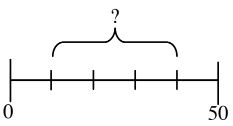
a.



b.



c.



1.2.6 What if there are multiple outcomes?

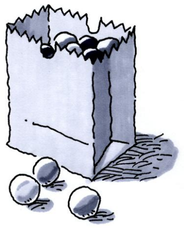
Compound Probability

So far in this section, you have worked with probabilities where there is one specific outcome that is desired. Today you will investigate probabilities of **compound events**, that is, events with combinations of outcomes. Today you will be interested in finding the probability that **either** one of the events **or** the other event occurs. In Chapter 4 you will consider the probability that **both** one event **and** another event occur. Think about these questions as you work with your study team:

How is this probability related to the probability of a single event?

Either what event, **or** what other event, are we interested in?

Does our answer make sense?



1-116. A bag holds 40 marbles, and each marble is either yellow or green. If you reach into the bag and randomly pull out a marble, you have an 80% chance of pulling out a yellow marble.

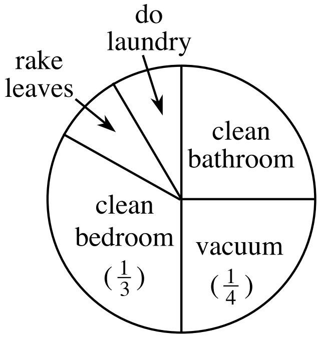
a. How many of the marbles are green? Show how you know.

b. How could you add or remove marbles in the original bag to change the probability of drawing a green marble to 50%?

c. **Additional Challenge:** Design a new bag of marbles that contains at least 50 marbles that are yellow, green, or red. How many of each color should you put in the bag to create a 25% probability of drawing red? Is there more than one possible way?

1-117. Thomas helps around the house by doing one chore after school. Each day Thomas and his aunt use the spinner at right to decide which chore he will do. Here is what Thomas knows:

- The sections on the spinner for "rake leaves" and "do laundry" are the same size.
- The sections for "clean bathroom" and "vacuum" are equal in size and together make up half the spinner.



a. What is the probability that Thomas will spin "do laundry"?

b. Thomas hates to clean the bathroom. When he spins the spinner, what is the probability that it will not point to "clean bathroom"? Explain how you found this answer.

1-118. Thomas' aunt hopes that he will **either** spin "clean bedroom" **or** "rake leaves" today.

a. What is P(clean bedroom)? What is P(rake leaves)?

b. Spinning either chore would make Thomas' aunt happy. With your study team, discuss the questions below and record your team's answers. Be sure to justify your conclusions.

- What is the probability that he will spin either one of the chores?
- How can you write the two outcomes as a single probability?

1-119. Ms. Nguyen lets her students borrow pens and pencils on days when they have a quiz. She has a paper bag with hundreds of wooden pencils, mechanical pencils, and blue pens in it.

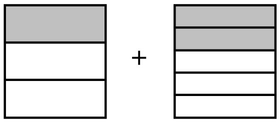
Stuart forgot his pencil, and it is quiz day! Ms. Nguyen tells him that one out of every three students who reaches into the bag pulls out a wooden pencil. Two out of every five students pull out a mechanical pencil. The rest of the students pull out a blue pen.

a. If Stuart reaches into the bag without looking, is it more likely that he will choose a wooden pencil or a mechanical pencil? Justify your thinking.

b. How can you describe the probability that Stuart will pull out some kind of pencil—**either** a wooden pencil **or** a mechanical pencil—by using the probabilities that you already know? Consider what you know about adding and subtracting fractions and see if you already have a strategy to write this probability as a single number.

1-120. Felicia was trying to find the probability that she would pull **either** a wooden pencil **or** a mechanical pencil out of Ms. Nguyen's bag from problem 1-119. *"I think I need to combine the probability that I will get a wooden pencil with the probability that I will get a mechanical pencil,"* she said. She set up this expression and drew a picture:

$$\frac{1}{3}+\frac{2}{5}$$



a. Felicia wondered if she could add the parts. Is the sum $\frac{3}{8}$? Why or why not?

b. Discuss with your team how Felicia could change the way she writes each fraction so that she can add them easily. Be ready to explain your reasoning. Then, find the sum.

1-121. Steve shuffles a standard deck of 52 playing cards and starts to turn them over one at a time. The first three cards he turns over are an ace, a four, and a jack.

a. How many cards are left in the deck?

b. How many of the remaining cards are aces?

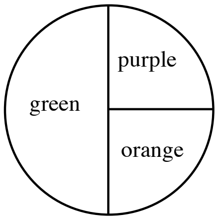
c. What is the probability that the fourth card will be an ace?

d. Instead of getting an ace, he gets a two as the fourth card. The fifth card is a five. What is the probability that the next card will be a king?

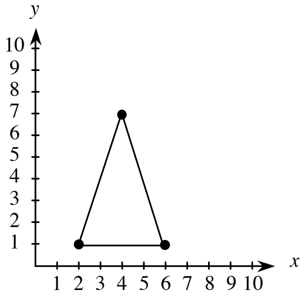
Review & Preview

1-122. What is the probability of getting either blue or green on a spinner that is $\frac{3}{10}$ green and $\frac{1}{5}$ blue? Show your work.

1-123. If you were to spin the spinner at right, what would be the probability of landing on green or purple? Explain how you know.



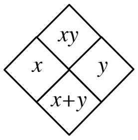
1-124. Locate the coordinates of the three highlighted points on the graph of the triangle below and write them as ordered pairs (x, y).

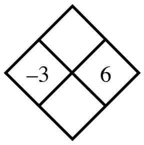


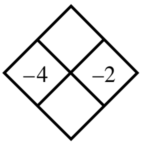
1-125. Draw an example of each of the following shapes. Refer to the glossary in the back of the book if you need help.

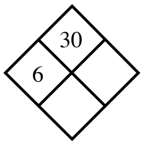
a. rectangle
b. square
c. parallelogram
d. trapezoid
e. scalene triangle
f. right triangle

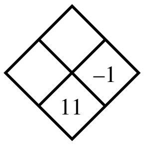
1-126. Copy and complete each of the Diamond Problems below. The pattern used in the Diamond Problems is shown at right.



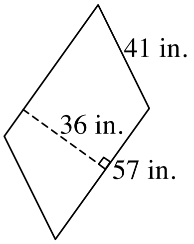
a.


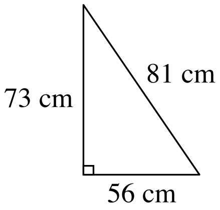
b.


c.


d.


1-127. Find the area and perimeter of each figure.

a.


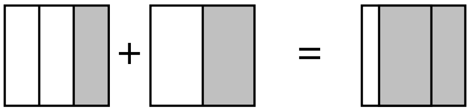
b.


1.2.7 How do I add outcomes?

Fraction Addition

When thinking about the probability of **either** one event **or** another occurring, you added fractions. Today you will investigate fraction addition further.

1-128. Justin is working on a new problem: $\frac{1}{3}+\frac{1}{2}$. He drew the picture below to represent the problem:



a. Could you name Justin's sum as a single fraction? Would he be correct to name it $\frac{2}{3}$?

b. Kermit sees Justin's work, and says, *"You need to make all of the pieces the same size before you try to write the sum. You can use a Giant One to rewrite* $\frac{1}{3}$ *and* $\frac{1}{2}$ *each as sixths."* He writes:

$$\frac{1}{3}\cdot\boxed{\frac{\ }{\ }}=\frac{\ }{6} \qquad \frac{1}{2}\cdot\boxed{\frac{\ }{\ }}=\frac{\ }{6}$$

Finish Kermit's work to rewrite each fraction so that its denominator is six.

c. What is the sum of the new fractions? Fix Justin's picture or draw a new diagram to explain why your answer makes sense.

d. Discuss with your team why Kermit might have chosen to rewrite each fraction as sixths. Could he have used a different number? Be ready to share your thinking.

1-129. Before we can write the sum of two fractions, they must be rewritten so that both wholes have an equal number of parts. In other words, they must be rewritten to have a **common denominator**. Three students were adding the fractions $\frac{1}{4}+\frac{5}{12}$ below, and they each found a different common denominator:

Lily:	Armando:	Josue:
$\frac{1}{4}+\frac{5}{12}$	$\frac{1}{4}+\frac{5}{12}$	$\frac{1}{4}+\frac{5}{12}$
$\frac{12}{48}+\frac{20}{48}$	$\frac{3}{12}+\frac{5}{12}$	$\frac{6}{24}+\frac{10}{24}$
$\frac{32}{48}$	$\frac{8}{12}$	$\frac{16}{24}$

a. Who is correct? Can any number be a common denominator? Explain your reasoning.

b. Which of their common denominators is easiest to work with? Why?

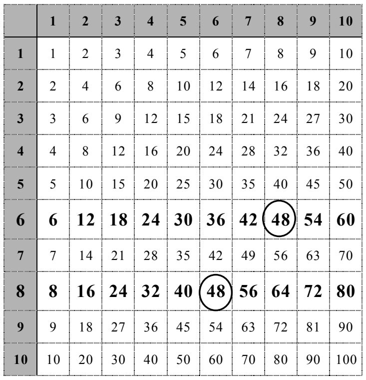
c. Find at least three different common denominators you could use to add $\frac{3}{10}+\frac{1}{5}$.

1-130. Vu needs to add $\frac{3}{8}+\frac{5}{6}$. He knows that he can rewrite each fraction as a portion of 48 because 6 and 8 are each factors of 48 in his multiplication table. *"Is there a smaller number that could work as a common denominator?"* he wonders.

a. Where can you look in the multiplication table to see if 6 and 8 are each factors of a number less than 48?

b. Use the multiplication table on the Lesson 1.2.7 Resource Page to find other number(s) you could use as a common denominator to add $\frac{3}{8}$ and $\frac{5}{6}$.

c. If Vu's next problem is to add $\frac{5}{4}+\frac{7}{10}$, use the multiplication table to find the smallest number you could use as a common denominator, called the **lowest common denominator**. Then, find the sum.



	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
1	1	2	3	4	5	6	7	8	9	10
2	2	4	6	8	10	12	14	16	18	20
3	3	6	9	12	15	18	21	24	27	30
4	4	8	12	16	20	24	28	32	36	40
5	5	10	15	20	25	30	35	40	45	50
6	**6**	**12**	**18**	**24**	**30**	**36**	**42**	**48**	**54**	**60**
7	7	14	21	28	35	42	49	56	63	70
8	**8**	**16**	**24**	**32**	**40**	**48**	**56**	**64**	**72**	**80**
9	9	18	27	36	45	54	63	72	81	90
10	10	20	30	40	50	60	70	80	90	100

1-131. One study team is trying to find the sum $\frac{5}{6}+\frac{3}{4}+\frac{1}{5}$.

a. They first tried to use 24 as a common denominator. Why does 24 not work as a common denominator?

b. Find a common denominator for this problem and explain how you found it.

c. With your study team, describe how factors and multiples can help you to find a common denominator.

1-132. With your team, find each sum. You may want to use the multiplication table to help you find common denominators.

a. $\frac{1}{4}+\frac{3}{5}$ b. $\frac{2}{7}+\frac{2}{3}$ c. $\frac{5}{8}+\frac{1}{2}$ d. $\frac{2}{3}+\frac{1}{2}+\frac{1}{6}$

1-133. THE POWER OF PRIME NUMBERS

Additional Challenge: Prime numbers are numbers that have exactly two factors, namely, one and themselves. Said another way, prime numbers can only be divided evenly by themselves and one (except for 1, which is not prime). Every number that is not prime can be rewritten as a product of **prime factors**. For example, $20 = 2 \cdot 2 \cdot 5$ or $36 = 2 \cdot 2 \cdot 3 \cdot 3$. Prime factors can be used to build common denominators for fractions.

a. What are the prime factors of each denominator in the sum $\frac{5}{6}+\frac{3}{4}+\frac{1}{5}$ (from problem 1-131)?

b. What are the prime factors of the common denominator you found in part (b) of problem 1-131?

c. What do the lists of factors you made in parts (a) and (b) have in common?

d. How could the prime factors of each denominator in a pair of fractions help you find a common denominator for those fractions? Explain your thinking.

Review & Preview

1-134. For each part below, find a Giant One that will multiply the fraction on the left side of the equation to create the equivalent fraction on the right side of the equation. Then complete any other missing information.

a. $\frac{5}{6} \cdot \boxed{\frac{\ }{\ }} = \frac{15}{\ }$ b. $\frac{\ }{3} \cdot \boxed{\frac{\ }{\ }} = \frac{4}{6}$

c. $\frac{\ }{2} \cdot \boxed{\frac{\ }{\ }} = \frac{8}{16}$ d. $\frac{\ }{4} \cdot \boxed{\frac{\ }{\ }} = \frac{6}{24}$

1-135. Fareed wants to add $\frac{1}{4}+\frac{5}{8}$.

a. Add the fractions by using a Giant One to create a common denominator.

b. How can factors help you find a common denominator?

1-136. Marisa and Mario were visiting the carnival when they noticed a few number relationships. They made them into brainteasers for you.

a. If three-tenths of the visitors were adults and there were 100 visitors, how many visitors were adults?

b. Five-eighths of the prizes at the Giant Spin were dolls. If there were 64 prizes, how many prizes were not dolls?

1-137. Order these numbers from least to greatest.

$$\frac{1}{2} \quad 1.1 \quad \frac{5}{3} \quad 2 \quad 0 \quad 0.4 \quad -2 \quad \frac{5}{8}$$

1-138. Find each sum.

a. $\frac{3}{5}+\frac{1}{3}$ b. $\frac{5}{7}+\frac{1}{2}$ c. $\frac{1}{6}+\frac{2}{8}$

1-139. Kirk is helping his grandparents set up their new portable music players. His grandmother, Maude, has 1 jazz album, 2 country-western albums, and 5 heavy-metal albums. Kirk's grandfather, Claude, has 3 classical albums, 2 rap albums, and 7 heavy-metal albums.

If Kirk's grandparents' portable music players are on random shuffle mode, who has the greater chance of listening to a heavy-metal album? Explain how you know.

1.2.8 What part is missing?

Subtracting Probabilities

In the previous lesson, you calculated probabilities for single events and combinations of events when **either** one outcome **or** the other is desirable. You have also compared those probabilities to decide which outcomes are most likely. In this lesson, you will work with your study team to apply your knowledge of fractions to represent and calculate probabilities of a variety of events. As you work today, ask each other these questions to focus your discussion:

How can probabilities be combined?

How can the answer be rewritten in a different form?

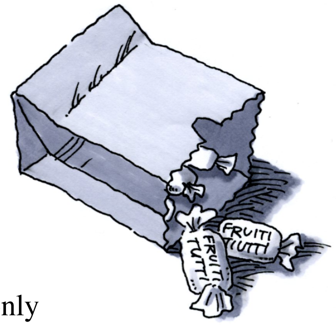
1-140. Eustice is adding fractions. Here is his work:

$$\frac{1}{2}+\frac{2}{4}=\frac{3}{6}$$

a. How do you think Eustice got his answer? Does his answer make sense? Discuss Eustice's work with your team, and then explain your reasoning on your paper with diagrams and words.

b. Help Eustice find the correct answer.

1-141. Lindsay has a paper bag full of Fruiti Tutti Chews in three different fruit flavors. She says, *"If you reach into the bag, you have a $\frac{1}{3}$ chance of pulling out a Killer Kiwi. There is a $\frac{3}{5}$ chance that you will get Crazy Coconut."* If you reach into the bag:



a. What is P(coconut or kiwi)?

b. Does there have to be another flavor in the bag? How can you tell? If so, assuming that there is only one other flavor, what is the probability of getting that flavor?

c. How many candies might Lindsay have in the bag? Is there more than one possibility? Assume that all candies in the bag are whole candies.

1-142. Lyle asked for a challenge problem, and his teacher gave him this one:

There is a $\frac{2}{7}$ chance of drawing a red marble out of a bag of marbles. If the probability of drawing a red or a blue marble is $\frac{2}{3}$, what is the probability of drawing a blue marble?

When Lyle's teammates saw the challenge he was working on, they each had a different idea for how he should start.

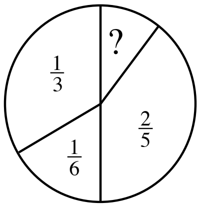
- Mayra suggested that he start by rewriting $\frac{2}{3}$ and $\frac{2}{7}$ so that they have a common denominator.
- Kenessa disagreed. *"You want to set up a subtraction problem,"* she said. She wrote the expression $\frac{2}{3}-\frac{2}{7}$ to show the number Lyle needs to find.
- Darren had a different idea. *"The probability that you would draw either red or blue is the same as the probability of drawing red added to the probability of drawing blue,"* he said. He wrote this expression:

$$\frac{2}{7}+?=\frac{2}{3}$$

a. Which way would you suggest Lyle start working to find P(blue)?
  - Discuss your ideas with your study team.
  - Write a clear explanation of which method you recommend and why.
  - Include your ideas about whether the other strategies will solve the problem and why you did not choose them.

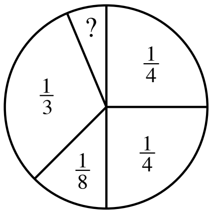
b. What is the probability of drawing a blue marble? Explain your reasoning.

1-143. Louise is playing a game, but the spinner is incomplete. Each section of the spinner is labeled with the probability of spinning it. What fraction is missing?



1-144. What math operations can you use to show your reasoning in problems 1-142 and 1-143? Discuss your strategies with your study team. Try to find more than one way to explain how you found each missing piece.

1-145. Charles found a spinner his teacher was making. He knew that if he put his problem-solving skills to use, he would be able to figure out the missing piece and finish the spinner for his teacher.



a. Write an expression for the problem Charles needs to solve. Is there more than one way to do this?

b. Find the solution.

1-146. Work with your study team to invent two new spinners with missing parts, like the spinners in problems 1-143 and 1-145. Then,

- Find the missing piece on your own paper.
- On a new sheet of paper, draw and label your spinner.
- Trade problems with another study team and find the missing parts of their spinners while they find yours.

1-147. LEARNING LOG

In your Learning Log, describe the process you use to add or subtract fractions when the probabilities have different denominators. Create an example and show your solution with pictures and number expressions. Then, answer the questions:

- How do you rewrite the fractions to represent pieces that are the same size?
- How do you identify what common denominator to use?

Label these notes “Adding Probabilities with Unlike Denominators,” and include today’s date.

MATH NOTES

METHODS AND MEANINGS

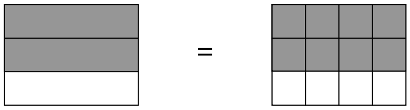
Equivalent Fractions

Fractions that are equal, but written in different forms, are called **equivalent fractions**. Rewriting a fraction in an equivalent form is useful when you want to compare two fractions or when you want to combine portions that are divided into different size pieces.

A Giant One is a useful tool to create an equivalent fraction. To rewrite a fraction in a different form, multiply the original fraction by a fraction equivalent to 1. For example:

$$\frac{2}{3}\cdot\boxed{\frac{4}{4}}=\frac{2\cdot4}{3\cdot4}=\frac{8}{12}$$

A picture can also demonstrate that these two fractions are equivalent:



Review & Preview

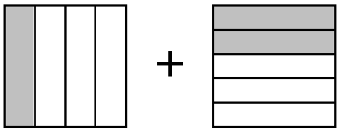
1-148. Rewrite each fraction in at least 2 different ways.

a. $\frac{6}{9}$ b. $\frac{11}{12}$ c. $\frac{3}{8}$ d. $\frac{10}{7}$

1-149. Jonathan measured two cups of flour into a bowl on the counter. Then he spilled part of it, and now there is only $\frac{3}{8}$ cup left. How much did he spill?

1-150. Mario is trying to explain to Marissa how to add fractions. He has written the work at right.

$\frac{1}{4}+\frac{2}{5}$



Step 1: $\frac{1}{4}\cdot\boxed{\frac{5}{5}}=\frac{5}{20}$ $\frac{2}{5}\cdot\boxed{\frac{4}{4}}=\frac{8}{20}$

a. Why did Mario start by rewriting the fractions using a Giant One? Why did he choose 20 as his new denominator?

b. Finish the problem. Explain each step you would take, and why.

1-151. Maggie is making a recipe that produces 18 muffins.

a. There are 12 people in Maggie's book club. If the muffins are divided evenly among each person, how much will each person get? Explain your thinking.

b. If Maggie wanted to divide the muffins evenly between the 36 students in her class, how much muffin would each person get? Explain your thinking.

1-152. **Multiple Choice:** If the probability of getting a particular result in an experiment is 75.3%, what is the probability of *not* getting that result? Explain your choice.

A. $75.3\% + 100\%$

B. $75.3\% - 100\%$

C. $100\% - 75.3\%$

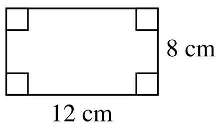
D. $\frac{1}{75.3\%}$

1-153. Throughout this book, key problems have been selected as "checkpoints." Each checkpoint problem is marked with an icon like the one at left. These checkpoint problems are provided so that you can check to be sure you are building skills at the expected level. When you have trouble with checkpoint problems, refer to the review materials and practice problems that are available in the "Checkpoint Materials" section at the back of your book.

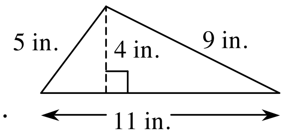
This problem is a checkpoint for area and perimeter of polygons. It will be referred to as Checkpoint 1.

For each figure below, find the area and the perimeter.

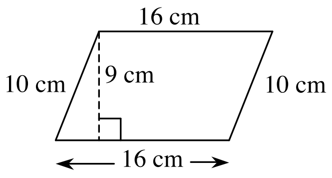
a.



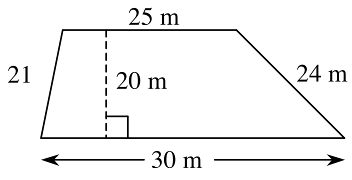
b.



c.



d.



Check your answers by referring to the Checkpoint 1 materials located at the back of your book. If you needed help solving these problems correctly, then you need more practice with computing area and perimeter. Review the Checkpoint 1 materials and try the practice problems. Also, consider getting help outside of class time. From this point on, you will be expected to do problems like these quickly and easily.

Chapter 1 Closure   What have I learned?

Reflection and Synthesis

The activities below offer you a chance to reflect about what you have learned during this chapter. As you work, look for concepts that you feel very comfortable with, ideas that you would like to learn more about, and topics you need more help with. Look for connections between ideas as well as connections with material you learned previously.

① SUMMARIZING MY UNDERSTANDING

This section gives you an opportunity to show what you know about certain math topics or ideas. Your teacher will assign you an activity to review the concepts you have studied in this course.

② WHAT HAVE I LEARNED?

Working the problems in this section will help you to evaluate which types of problems you feel comfortable with and which ones you need more help with. This section will appear at the end of every chapter to help you check your understanding.

Solve each problem as completely as you can. The table at the end of this closure section has answers to these problems. It also tells you where you can find additional help and practice on problems like them.

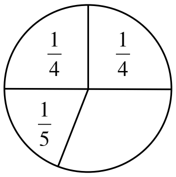
CL 1-154. Vanson has a bunch of boxes that are all the same. He stacked four boxes, measured the stack, and found that it was three feet high.

a. How high will a stack of 20 of these boxes be?

b. The ceiling in the room where Vanson is working is just a little more than 9 feet high. He wants to stack boxes from the floor to the ceiling. How many boxes will fit in one stack?

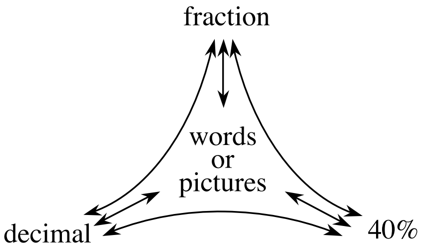
CL 1-155. Aiden is drawing coins from a bag containing 6 pennies, 3 nickels, 4 dimes, and 2 quarters. What is the probability that he will draw a penny?

CL 1-156. Tuan is playing a game, but the spinner is incomplete. If the numbers in the sections of the spinner represent the probabilities of spinning each section, help him figure out the fraction for the missing section of the spinner.

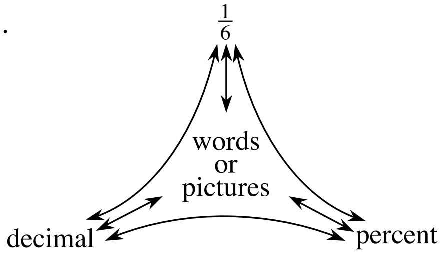


CL 1-157. Complete each portions web.

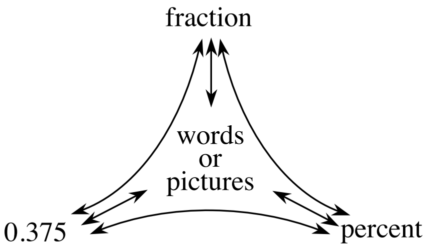
a.



b.



c.



CL 1-158. Add $\frac{1}{6} + \frac{1}{2}$. Show all of your steps.

CL 1-159. Write "theoretical" or "experimental" to describe the following situations.

a. The chance of rolling a sum of three with two number cubes is $\frac{1}{18}$.

b. I drew five cards out of a deck and got clubs three times.

c. I bought six raffle tickets and did not win anything.

d. Based on a mathematical model, the chance of a flood next year is 1.2%.

CL 1-160. The county fair prize wheel has equally spaced sections with the following colors: one is golden, two are silver, three are green, four are blue, six are red, and nine are yellow.

a. What is the probability of landing on gold? Give your answer as a fraction and as a percent.

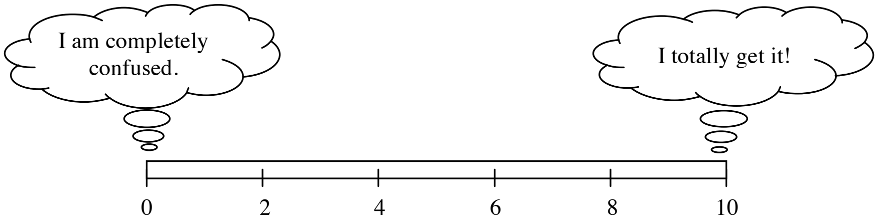
b. If the probability of landing on yellow is 36%, what is the probability of not landing on yellow?

c. If the wheel is spun 100 times, how many times would you expect to land on silver?

CL 1-161. Kimberly is playing "Guess My Number." Her clue is, *"When I triple my number and subtract 7, I get 83."* Find Kimberly's number and explain how you know your answer is correct.

CL 1-162. For each of the problems above, do the following:

- Draw a bar or number line that represents 0 to 10.



- Color or shade in a portion of the bar that represents your level of understanding and comfort with completing that problem on your own.

If any of your bars are less than a 5, choose *one* of those problems and complete one of the following tasks:

- Write two questions that you would like to ask about that problem.
- Brainstorm two things that you DO know about that type of problem.

If all of your bars are a 5 or above, choose *one* of those problems and do one of these tasks:

- Write two questions you might ask or hints you might give to a student who was stuck on the problem.
- Make a new problem that is similar and more challenging than that problem and solve it.

③ WHAT TOOLS CAN I USE?

You have several tools and references available to help support your learning – your teacher, your study team, your math book, and your Toolkit, to name only a few. At the end of each chapter you will have an opportunity to review your Toolkit for completeness as well as to revise or update it to better reflect your current understanding of big ideas.

The main elements of your Toolkit should be your Learning Logs, Math Notes, and the vocabulary used in this chapter. Math words that are new to this chapter appear in bold in the text. Refer to the lists provided below and follow your teacher's instructions to revise your Toolkit, which will help make it a useful reference for you as you complete this chapter and prepare to begin the next one.

Learning Log Entries

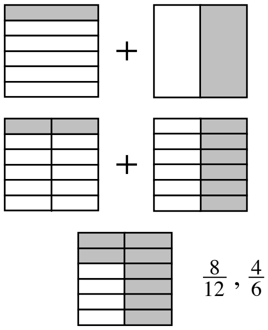
- Lesson 1.1.4 – Beginning to Think About Proportional Relationships
- Lesson 1.2.1 – Probability
- Lesson 1.2.2 – Theoretical and Experimental Probabilities
- Lesson 1.2.5 – Comparing Probabilities
- Lesson 1.2.8 – Adding Probabilities with Unlike Denominators

Math Notes

- Lesson 1.1.2 – Perimeter and Area
- Lesson 1.1.3 – Mean
- Lesson 1.1.4 – Median
- Lesson 1.1.5 – Representations of Portions
- Lesson 1.2.2 – Scaling a Graph
- Lesson 1.2.3 – Probability Vocabulary and Definitions
- Lesson 1.2.5 – Additive and Multiplicative Identities
- Lesson 1.2.8 – Equivalent Fractions

Answers and Support for Closure Problems
What Have I Learned?

Note: MN = Math Note, LL = Learning Log

Problem	Solution	Need Help?	More Practice
CL 1-154.	a. 15 feet b. 12 boxes will fit in one stack	Lesson 1.1.3 MN: 1.1.2	Problems 1-37, 1-38, 1-40, and 1-41
CL 1-155.	$\frac{6}{15}$ or $\frac{2}{5}$	Lesson 1.2.1 MN: 1.2.2	Problems 1-54, 1-55, 1-56, 1-60, and 1-62
CL 1-156.	$\frac{3}{10}$	Lesson 1.2.2 MN: 1.2.2	Problems 1-66, 1-67, 1-68, and 1-69
CL 1-157.	a. Two fifths, 0.4, 40% b. One sixth, $0.1\overline{6}$, 16.67% c. Three eighths, 0.375, 37.5%	Lesson 1.1.5 MN: 1.1.5	Problems 1-46, 1-48, 1-49 and 1-63
CL 1-158.	 $\frac{8}{12}$, $\frac{4}{6}$, or $\frac{2}{3}$	Lesson 1.2.7 MN: 1.2.8 LL: 1.2.8	Problems 1-128, 1-129, 1-130, 1-131, 1-132, 1-135, and 1-138
CL 1-159.	a. Theoretical b. Experimental c. Experimental d. Theoretical	Lessons 1.2.1 and 1.2.2 MN: 1.2.3 LL: 1.2.1 and 1.2.2	Problems 1-58, 1-62, and 1-67
CL 1-160.	a. $\frac{1}{25} = 4\%$ b. 64% c. 8	Lessons 1.2.1 and 1.2.2 LL: 1.2.1	Problems 1-54, 1-55, 1-56, and 1-60
CL 1-161.	Kimberly's number is 30. $30 \cdot 3 - 7 = 83$	Lesson 1.1.3	Problems 1-16, 1-17, and 1-18